diluted Cr^{3+} ions in corundum,[21] where Cr^{3+} substitutes for Al^{3+} without change of site symmetry,

$$G_{44} = 1.85 \text{ cm}^{-1}, \quad G_{46} = 1.55 \text{ cm}^{-1} .$$

The present values are of the same order of magnitude.

The theoretical part of this paper can be transposed *mutatis mutandis* to other antiferromagnetic crystals of different symmetries provided the magnetization easy axis is an acoustical axis. This includes, for instance, the case of MnF_2.

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[1]C. Kittel, Phys. Rev. 110, 836 (1958).

[2]H. Matthew and R. C. Le Craw, Phys. Rev. Letters 8, 397 (1962).

[3]R. Guermeur, J. Joffrin, A. Levelut, and J. Penné, Solid State Commun. 5, 369 (1967). The authors take this opportunity to point out that, when a parabolic law is supposed for the internal magnetic field along the rod, their result must be corrected to $b_2 = 12.9 \times 10^6$ erg/cm^3 ($T = 4.2$ °K, $\nu = 10^{10}$ Hz). Value of H_A can be deduced: $H_A = -220$ G.

[4]R. Guermeur, J. Joffrin, A. Levelut, and J. Penné, Solid State Commun. 6, 519 (1968).

[5]J. R. Boyd and J. D. Gavenda, Phys. Rev. 152, 645 (1966).

[6]M. Boiteux, P. Doussineau, B. Ferry, J. Joffrin, and A. Levelut, Solid State Commun. 8, 1609 (1970).

[7]S. Foner, Phys. Rev. 130, 183 (1963).

[8]Y. Shapira, Phys. Rev. 187, 734 (1969).

[9]Y. L. Wang and H. B. Callen, J. Phys. Chem. Solids 25, 1459 (1964).

[10]S. V. Peletminskii, Zh. Eksperim. i Teor. Fiz. 37, 452 (1959) [Sov. Phys. JETP 37, 321 (1960)].

[11]M. A. Savchenco, Fiz. Tverd. Tela 6, 666 (1964) [Sov. Phys. Solid State 6, 864 (1964)].

[12]Y. Shapira and J. Zak, Phys. Rev. 170, 503 (1970).

[13]R. L. Melcher, J. Appl. Phys. 41, 1412 (1970).

[14]R. L. Melcher, Phys. Rev. Letters 25, 235 (1970).

[15]M. Boiteux, P. Doussineau, B. Ferry, and U. T. Höchli, J. Phys. Radium 32, 496 (1971).

[16]U. N. Upadhyaya and K. P. Shina, Phys. Rev. 130, 939 (1963).

[17]S. A. Al'tshuler, B. I. Kochelaev, and A. M. Leushin, Usp. Fiz. Nauk 75, 459 (1961) [Sov. Phys. Usp. 4, 880 (1962)].

[18]W. I. Dobrov, Phys. Rev. 134, A734 (1964).

[19]C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1963).

[20]E. J. Samuelsen, M. T. Hutchings, and G. Shirane, Physica 48, 13 (1970).

[21]P. L. Donoho, Phys. Rev. 133, 1080 (1964).

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Scattering of Polarized Electrons from Magnetic Materials

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Cross-section formulas for large-angle (e, e) and for coincidence (e, $2e$) scattering of fast electrons from gaseous and solid targets are derived. It is shown that the momentum and spin-density distribution of the bound electrons can be obtained from single (e, e) scattering of polarized electrons. It is also shown that the electronic spin- and energy-dependent momentum-density distribution can be obtained from (e, $2e$) scattering of polarized electrons. An (e, $2e$) experiment with unpolarized electrons gives the energy-dependent momentum distribution. A comparison is made with related methods, like Compton scattering, and the feasibility of the suggested experiments is discussed.

I. INTRODUCTION

Recently, several papers[1–3,5–10] dealing with electron-momentum-density distributions in atoms, molecules, liquids, and solid materials have appeared. Three experimental techniques have been used for obtaining these distributions (a fourth less straightforward method is briefly discussed at the end of this paper).

The first method involves large-angle Compton scattering of x rays. Theoretical and experimental aspects of this method have been described in much detail by Eisenberger and Platzman,[1] Eisenberger,[2] and Currat *et al.*[3] The momentum distribution is obtained in this case from the frequency distribution of the x rays scattered through a fixed angle. The same information can be obtained from the angular distribution of x rays of a suitable frequency.

The second method involves large-angle and large-energy-loss scattering of fast electrons from gases or solids (reflection or transmission through thin films). This technique was applied already more than 30 years ago[4] for gaseous He and H_2. The momentum distribution is obtained either from the energy distribution[4] of the electrons scattered through a particular angle, or from the angular distribution[5] of the electrons scattered with a particular energy loss.

The third method is related to the second one in that it involves scattering of fast electrons. It is different in that scattered and ejected electrons are detected in coincidence. Such $(e, 2e)$ measurements have been reported already by Amaldi *et al.*[6] Also, several theoretical papers have appeared on this subject.[7–10]

The coincidence technique allows one[6–10] to select a particular energy band in the target and to obtain the energy-dependent electronic-momentum-density distribution $\rho(\vec{k}, E)$, while the first two methods only yield $\rho(\vec{k})$, where $\rho(\vec{k}) = \int \rho(\vec{k}, E)\, dE$. This more detailed information is obtained at the cost of orders-of-magnitude lower count rates and correspondingly longer measuring times.

The theory used to describe large-angle Compton scattering[1,2] is closely related to the one used for large-angle electron scattering.[5–10] The plane-wave approximation is made in describing the fast-ejected (recoiled) electron, and the mathematical techniques employed in both theories are almost identical to one another.[11]

Recently, Platzman and Tzoar[12] introduced a new concept into the theory of Compton scattering: the determination of the spin-dependent momentum distribution of electrons in ferromagnetic materials by (incoherent) Compton scattering of polarized x rays. In the present paper we show that the spin-dependent electronic-momentum-density distribution can be obtained also from scattering of polarized electrons. It is shown that the spin effect is a large first-order effect in electron scattering, provided appropriate scattering (and ejection) angles are used. In Compton scattering it is a relativistic effect, which is large only if the x-ray energies are comparable to the rest mass of the electron. Furthermore, it is shown that the coincidence technique can be used again to determine in this case the spin- and energy-dependent momentum distribution $\rho(\vec{k}, E, \theta)$.

II. TRANSITION MATRIX ELEMENTS

In this paper we use a_0, the Bohr radius, as unit of length, give cross sections in units of a_0^2, represent momenta by their wave vectors $\vec{k}$ in units of a_0^{-1}, and give energies in Rydberg units.

The Born transition matrix element for single scattering of an electron from an N-electron system (atom, molecule, or solid) may be written as[7,9]

$$T = \sum_j T_j = (2\pi)^{-3} \sum_j \langle e^{i\vec{k}_s\cdot\vec{r}_0}\, \psi_f(\vec{r}_1 \cdots \vec{r}_N) \times | 1/r_{0j} | e^{i\vec{k}_i\cdot\vec{r}_0}\, \psi_i(\vec{r}_1 \cdots \vec{r}_N) \rangle \; , \quad (1)$$

where $\vec{r}_0$ and $\vec{r}_1$ to $\vec{r}_N$ are the position coordinates of the incident electron and of the bound electrons, respectively, $\vec{k}_i$ and $\vec{k}_s$ are the initial and final momenta of the primary electron, and ψ_i and ψ_f are the initial- and final-state wave functions of the N-electron system. Integration over $\vec{r}_0$ yields

$$T_j = (1/2\pi^2 K^2) \langle \psi_f(\vec{r}_1 \cdots \vec{r}_N) | e^{i\vec{K}\cdot\vec{r}_j} | \psi_i(\vec{r}_1 \cdots \vec{r}_N) \rangle \; , \quad (2)$$

where $\vec{K} = \vec{k}_i - \vec{k}_s$ is the momentum transfer. For very large K, $\exp i\vec{K}\cdot\vec{r}_j$ oscillates rapidly and T_j is very small unless ψ_f contains a term which oscillates in phase with $\exp i\vec{K}\cdot\vec{r}_j$. This situation is encountered in fast electron ejection, in which case ψ_f may be approximated by the product-type wave function

$$\psi_f(\vec{r}_1 \cdots \vec{r}_N) = (2\pi)^{-3/2} e^{i\vec{k}_e\cdot\vec{r}_j} \psi_f(\vec{r}_1 \cdots \vec{r}_{j-1}, \vec{r}_{j+1} \cdots \vec{r}_N) \; , \quad (3)$$

where the ejected electron with final momentum $\vec{k}_e$ is described by a plane wave, as is justified when k_e^2 very much exceeds the initial binding energy of the jth electron. For the initial-state wave function we may use the expansion

$$\psi_i(\vec{r}_1 \cdots \vec{r}_N) = \sum_l \varphi_l(\vec{r}_j) \psi_l(\vec{r}_1 \cdots \vec{r}_{j-1}, \vec{r}_{j+1} \cdots \vec{r}_N) \; , \quad (4)$$

where the ψ_1 represent a complete set of $N-1$ electron wave functions and the $\varphi_1(\vec{r}_j)$ are expansion coefficients, which itself may further be expanded into a complete set of one-electron wave functions $\varphi'_n(\vec{r}_j)$:

$$\varphi_l(\vec{r}_j) = \sum_n \langle i | l, n \rangle \varphi'_n(\vec{r}_j) \; , \quad (5)$$

where $\langle i | l, n \rangle$ are fractional parentage coefficients.[7] The latter expansion need not be used in this paper.

Substitution of (3) and (4) into (1) and transformation to momentum space yields

$$T_j = (1/2\pi^2 K^2) \langle \delta(\vec{k}_j + \vec{k}_i - \vec{k}_s - \vec{k}_e) | \phi_f(\vec{k}_j) \rangle \; , \quad (6)$$

where $\phi_f(\vec{k}_j)$ is the Fourier transform of $\varphi_f(\vec{r}_j)$. In Eq. (6), we may integrate over momentum space, with the result

$$T_j = (1/2\pi^2 K^2) \phi_j(\vec{k}_s + \vec{k}_e - \vec{k}_i) \; . \quad (7)$$

When the total wave functions are approximated by product-type wave functions (e.g., Slater determinants), then $\varphi_1(\vec{r}_j)$ and $\phi_j(\vec{k})$ directly represent the one-electron coordinate- and momentum-space wave functions of the bound electrons.

III. COINCIDENCE CROSS SECTIONS

From the known relationship between matrix elements and scattering cross sections it follows that

$$\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e) = (8\pi^4 k_s k_e / k_i) \left| \sum_j T_j \right|^2 \times \delta(k_i^2 - k_s^2 - k_e^2 - E) \; , \quad (8)$$

where $\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)$ is the differential cross section per unit energy interval about the energy loss $\epsilon = k_i^2 - k_s^2$, for scattering of the incident electron into a unit solid angle about $\hat{k}_s$ and for simultaneous ejection of one of the target electrons, bound with the "hole" energy E, into a unit solid angle about $\hat{k}_e$. The δ function is a consequence of energy conservation. For large momentum and large energy transfer, hence when $\langle \vec{K}\cdot(\vec{r}_j-\vec{r}_m)\rangle$ is large compared to unity and ϵ is large compared to the relevant binding energies of the target electrons, the scattering is an incoherent process and $|\sum_j T_j|^2$ may be replaced by $\sum_j |T_j|^2$. This simplification is also made in Refs. 1, 2, and 12. For some further information about conditions for which correlation terms $(T_j T_m^*)$ are negligible, see Refs. 13 and 14.

We have now arrived at the conclusion that large-angle and large-energy-loss scattering of a fast electron, resulting in the ejection of a fast electron, may be described as a binary collision between the two electrons. Because of the indistinguishability of these electrons, we have to include exchange and interference terms in the cross section formulas. For two-electron interactions it is well known[15] how this should be done. From Eqs. (7) and (8) we now find

$$\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)=\frac{2k_s k_e}{k_i}\sum_j |\phi_j(\vec{k}_s+\vec{k}_e-\vec{k}_i)|^2 \times\left(\frac{1}{K^4}+\frac{1}{S^4}-\frac{1+\cos\theta_{ij}}{K^2S^2}\right)\times\delta(k_i^2-k_s^2-k_e^2-E)\ , \qquad (9)$$

where θ_{ij} is the angle between the spin directions of the two interacting electrons and $\vec{S}=\vec{k}_i-\vec{k}_e$. Equation (9) implicitly assumes that the spin directions of the interacting electrons do not change throughout the collision.

In the case of unpolarized incident electrons and/or an isotropic spin distribution in the target, we have to average over θ_{ij} and the term $\cos\theta_{ij}$ vanishes. The $(e, 2e)$ experiments reported so far[6,16] and the theoretical papers on this subject have been confined to such unpolarized incident beams and isotropic targets.

From Eq. (9) it follows that spin anisotropies will show up most clearly when $S=K$. This situation is encountered when scattered and ejected electrons are detected at equal but opposite angles on either side of the primary beam, with symmetrically adjusted energy pass bands of the two analyzers. Equation (9) is then reduced to

$$\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)=\frac{2k_s k_e}{k_i K^4}\sum_j |\phi_j(\vec{k}_s+\vec{k}_e-\vec{k}_i)|^2 \times(1-\cos\theta_{ij})\delta(k_i^2-k_s^2-k_e^2-E)\ , \qquad (10)$$

and the term $1-\cos\theta_{ij}$ varies between 0 and 2, which implies that the spin effect is a significant first-order effect.

So far we have confined ourselves to targets containing only a finite number of bound electrons; $|\phi_j(\vec{k}_s+\vec{k}_e-\vec{k}_i)|^2$ represented the momentum distribution of the jth electron. $\sum_j |\phi_j(\vec{k}_s+\vec{k}_e-\vec{k}_i)|^2 \times\delta(k_i^2-k_s^2-k_e^2-E)$ represented the momentum distribution of the electrons bound with the energy E, and $\sum_j |\phi_j(\vec{k}_s+\vec{k}_e-\vec{k}_i)|^2(1-\cos\theta_{ij})\,\delta(k_i^2-k_s^2-k_e^2-E)$ represented the corresponding spin-dependent distribution.

For solid targets, we get the scattering cross section per unit volume by simply replacing the summations over a finite number of electrons by the corresponding density distributions. Instead of Eq. (9) we may write

$$\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)=\frac{2k_s k_e}{k_i}\,\rho_E(\vec{k}_s+\vec{k}_e-\vec{k}_i) \times\left(\frac{1}{K^4}+\frac{1}{S^4}-\frac{1+\langle\cos\theta_i\rangle_E}{K^2S^2}\right)\times\delta(k_i^2-k_s^2-k_e^2-E)\ , \qquad (11)$$

and Eq. (10) is replaced by

$$\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)=\frac{2k_s k_e}{k_i K^4}\,\rho_E(\vec{k}_s+\vec{k}_e-\vec{k}_i) \times(1-\langle\cos\theta_i\rangle_E)\delta(k_i^2-k_s^2-k_e^2-E)\ , \qquad (12)$$

where $\rho_E(\vec{k}_s+\vec{k}_e-\vec{k}_i)=\rho_E(\vec{k})$ is the $\rho(\vec{k}, E)$ of the introduction. From Eq. (12) it follows that the spin- and energy-dependent momentum distribution $\rho(\vec{k}, E, \theta)$ is obtained by multiplying the measured $\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e)$ by $k_i K^4/2k_s k_e$, and by appropriately varying θ_i. When the variation in θ_i results from a rotation of the target, one may expect to find anisotropies caused by the spin distribution $(1-\langle\cos\theta_i\rangle_E)$ as well as[3] by anisotropies in $\rho_E(\vec{k}_s+\vec{k}_e-\vec{k}_i)$. The latter effect may be studied separately with an unpolarized electron beam.

IV. SINGLE-ELECTRON SCATTERING

The single- (e, e) scattering cross section $\sigma(\epsilon, \hat{k}_s)$ is obtained from Eq. (11) by integrating over (i) all hole energies E and (ii) all angles of ejection of the second electron. We find

$$\sigma(\epsilon, \hat{k}_s)=\frac{2k_s k_e}{k_i}\iint \rho_E(\vec{k}_s+\vec{k}_e-\vec{k}_i) \times\left(\frac{1}{K^4}+\frac{1}{S^4}-\frac{1+\langle\cos\theta_i\rangle_E}{K^2S^2}\right)\times\delta(k_i^2-k_s^2-k_e^2-E)\,dE\,d\hat{k}_e\ . \qquad (13)$$

Spin effects show up most pronouncedly when the first and the second term within the large parentheses of Eq. (13) give about equal contributions to the integral. For the weakly bound electrons in the solid, which may chiefly be responsible for the magnetization, $\rho_E(\vec{k})$ has its principal maximum near $k=0$ and is very small for large k. If we simultaneously require that $\vec{k}_s+\vec{k}_e\approx\vec{k}_i$, $K\approx S$, and $E\approx 0$, then $\vec{k}_i\cdot\vec{k}_s=k_s^2=\frac{1}{2}k_i^2$, which implies that the electron detector should be placed at an angle of 45° with respect to the primary beam, and its energy pass band should be adjusted to one-half of the primary energy. A deconvolution will further be necessary to obtain $\rho(\vec{k},\theta)=\int\rho(\vec{k},E,\theta)\,dE$ from the $\sigma(\epsilon,\hat{k}_s)$ measured (i) in a range of angles about 45° with respect to $\vec{k}_i$, or (ii) in a range of energies about $\frac{1}{2}k_i^2$. As mentioned before, one cannot distinguish in a single-scattering experiment between different binding energies. However, the weakly bound outer electrons occupy a much larger volume of coordinate space than the strongly bound inner electrons. Conversely, the outer electrons occupy a much smaller volume of momentum space and the $\rho(\vec{k},\theta)$ for small $\vec{k}$ corresponds chiefly to the outer electrons, while for large $\vec{k}$ it corresponds chiefly to the inner electrons.

V. COMPARISON WITH COMPTON AND NEUTRON SCATTERING

If we compare the electron scattering cross section $\sigma(\epsilon,\hat{k}_s)$ with the corresponding Compton scattering formulas of Refs. 1, 2, and 12, we see that our $\rho_E(\vec{k})$, with $\vec{k}=\vec{k}_s+\vec{k}_e-\vec{k}_i$, directly corresponds to the $|f(p_0)|^2$ in Eq. (20) of Ref. 1. Furthermore, if we assume (as was done in Refs. 1, 2, and 12) that the target electrons with initial momentum $\vec{k}$ are free and therefore have a binding energy $E=-k^2$, then $\delta(k_i^2-k_s^2-k_e^2-E)=\delta(\epsilon-K^2-2\vec{k}\cdot\vec{K})$ and we have reproduced the δ function encountered in Eq. (20) of Ref. 1 [see also Eqs. (11) and (14) of Ref. 12]. The spin effect in electron scattering is quite different from that in Compton scattering. In electron scattering it is a larger effect.

Platzman and Tzoar[12] have further discussed the coherent scattering of x rays, which process is analogous to neutron scattering. Coherent (diffractive) scattering is characterized by small values of $\vec{K}$ and ϵ; it involves either elastic scattering or collective small-ϵ excitations. Information is obtained about the form factors which are the Fourier transforms of the coordinate-space wave functions squared. In this paper we have confined ourselves to large-K and large-ϵ single-particle (electron) excitations. Information is obtained about the Fourier transforms of the coordinate-space wave functions ψ_i and not about $|\psi_i|^2$. Hence, there is hardly any overlap between diffractive scattering and large-K and large-ϵ scattering.

In coherent (diffractive) scattering of high-energy electrons (HEED), ϵ and K are also very small compared to k_i^2 and S. One can completely neglect exchange scattering and the HEED patterns for nonrelativistic (<40 keV) electrons are not affected by spin anisotropies. For relativistic electrons, additional spin terms are introduced in the cross-section formulas; it remains true, however, that the HEED patterns are unaffected by spin anisotropies.

An advantage of neutron and x-ray scattering as compared to electron scattering is the much larger penetration depth of neutrons and x rays in matter. An advantage of incoherent electron or Compton scattering as compared to diffractive neutron or x-ray scattering is that also noncrystalline materials can be studied.

VI. FEASIBILITY OF EXPERIMENT

In measuring scattering of fast electrons from magnetic targets, one wishes to keep deflections of the electrons in the magnetic field of the target as small as possible. It is helpful already that we confine ourselves to fast electrons (e.g., from 4 to 40 keV). If one uses small pieces or thin films of magnetic material, no further problems are to be expected in large-angle and large-energy-loss scattering of polarized electrons. The techniques of working with beams of polarized electrons and high-resolution electron-energy analyzers have been described already in much detail in literature.[17,18] The scattering cross sections are large enough to give quite acceptable count rates. A review about single scattering of polarized and unpolarized electrons from unpolarized and polarized atomic beams and from surfaces is given by Kessler.[17] Almost all of the work reported there involves elastic scattering and again hardly any overlap can be found with the processes discussed here.

Coincidence measurements are much more difficult to perform than single-scattering experiments. Firstly, since one is most interested in the $\rho(\vec{k},E,\theta)$ for small $\vec{k}$, $\vec{k}_e$ should lie in or very near to the plane formed by $\vec{k}_i$ and $\vec{k}_s$. Bulk materials cannot be studied therefore (either the recoiled or the scattered electron would penetrate into the material and lose its energy in multiple collisions). Coincidence measurements are practically possible only with thin (<1000–2000 Å) crystalline or noncrystalline films and fast (>10–40 keV) electrons. These limits arise from the condition that multiple scattering should be negligible. Multiple scattering may also smear out to some extent the energy and angular distribution of fast electrons scattered through about 45° from a "bulk" material (not from a thin film).

In an $(e,2e)$ experiment, electron-energy analyzers and polarized electron beams again do not present insurmountable problems. The major problem encountered[6,16] is the low count rates. This problem is now considered in more detail. Let N_C and I_0 be the number of coincidences per second and the incident electron current; then we have

$$N_C=(I_0/e)\sigma(\epsilon,E,\hat{k}_s,\hat{k}_e)l_c\Delta\hat{k}_s\,\Delta\hat{k}_e\Delta\epsilon\ , \qquad (14)$$

where e is the electron charge, l_c is the pathlength from which coincidence scattering is measured, $\Delta\hat{k}_s$ and $\Delta\hat{k}_e$ are the opening angles of the two detectors, and $\Delta\epsilon$ is the energy pass band. For $l_c \approx 2000$ (in units of a_0, hence about 1000 Å), $\Delta\hat{k}_s = \Delta\hat{k}_e \approx 3\times10^{-4}$ (rad), $\Delta\epsilon \approx 2\times10^{-2}$ (≈ 0.27 eV), and $I_0 \approx 10^{-8}$ A, we have

$$N_C \approx (2\times10^5)\,\sigma(\epsilon, E, \hat{k}_s, \hat{k}_e) \; . \qquad (15)$$

For 45° scattering, with $K\approx S$ and $k_s^2 \approx k_e^2 \approx \frac{1}{2}k_i^2$, we find, according to Eqs. (12) and (15),

$$N_C \approx (10^6/k_i^3)\rho_E(\vec{k}, \theta) \; . \qquad (16)$$

If we now assume that the energy band selected in the solid contains two electrons per eV, that the solid contains 3×10^{22} atom/cm^3, and that the primary electron energy is 20 keV, then

$$N_C \approx 2 f_E(k) \; , \qquad (17)$$

where $f_E(\vec{k})$ is the probability distribution of $\vec{k}$, which distribution is normalized to unity. For small E, $f_E(\vec{k})$ will be sharply peaked near $k=0$ and $f_E(\vec{k})$ may be of the order of $0.5-(0.5\times10^{-3})$. The corresponding count rates of $1-10^{-3}s^{-1}$ imply that a 5% accuracy is obtained with integration times exceeding 7 min to 100 h. Such integration times are quite common in nuclear physics. The coincidence experiment, although difficult and time consuming, should therefore be considered to be a possible experiment.

VII. CONCLUDING REMARKS

Throughout this paper we have described the incident and the two outgoing electrons by plane waves. This is justified only when the kinetic energies of these electrons very much exceed the relevant binding energies in the target. When one of the outgoing electrons is slow, the scattering cross sections are not proportional any more to the momentum density distributions and correlation terms are no longer negligible. The plane-wave approximation is the most essential approximation made in this paper.

A second, but well justified, approximation made is the one that the spin-flip cross sections are negligibly small for nonrelativistic electrons.[19]

Thirdly, after having paid so much attention to polarized electrons, we should note that $(e, 2e)$ experiments with unpolarized electrons, with the purpose of getting information about band structures in thin films, have not yet been reported in literature. These measurements are easier to perform than the ones with polarized electrons, because it is much simpler to get an "intense monoenergetic" unpolarized electron beam.

Further, we should note that the attempt has also been made[20,21] to obtain the momentum and spin density distribution $\rho(\vec{k}, \theta)$ from the angular correlation between the two γ quanta resulting from positron annihilation $(e^+e^-, \gamma\gamma)$. When fast positrons are incident on a (e.g., magnetic) material, they are first thermalized before annihilation takes place. The chief difficulty encountered is that the slow positrons polarize the surrounding electron gas,[20] which makes the interpretation of the experimental data much less straightforward than in x-ray and fast-electron scattering.

Finally, to clarify the position of the present work from a more general point of view, we add two references. The first one is to Vredevoe and de Wames,[22] who studied "magnetic scattering of electrons from crystals and polarization of the scattered beam." The relation to the present work is that exchange effects are also discussed. The effects considered by Vredevoe and de Wames differ from the ones dealt with here in that the former effects[22] are largest at small primary energies and small scattering angles. Also, a theoretical treatment[22] of scattering of slow electrons must include distortion effects and is of a far from trivial nature, in contrast to the situation encountered in large-angle scattering of fast electrons (this paper and Ref. 10). Secondly, we refer to Raether,[23] who has reviewed the experimental work on characteristic energy-loss measurements. A significant part of this sort of work is done with fast-electron beams and in this aspect it is related to our work. The differences are again that characteristic energy losses (usually between 0.1 and 300 eV) are much smaller than the energy transfers considered here (about 10 keV for a 20-keV primary beam) and that small (or zero) scattering angles are used for studying these small losses.

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[1]P. Eisenberger and P. M. Platzman, Phys. Rev. A 2, 415 (1970).

[2]P. Eisenberger, Phys. Rev. A 2, 1678 (1970).

[3]R. Currat, P. D. DeCicco, and R. Kaplow, Phys. Rev. B 3, 243 (1971).

[4]A. L. Hughes and M. M. Mann, Phys. Rev. 53, 50 (1937); A. L. Hughes and M. A. Starr, *ibid.* 54, 189 (1938).

[5]L. Vriens, in *Case Studies in Atomic Collision Physics* 1, edited by E. W. McDaniel and M. R. C. McDowell (North-Holland, Amsterdam, 1969), p. 335.

[6]U. Amaldi, E. Egidi, R. Marconero, and G. Pizzella, Rev. Sci. Instr. 40, 1001 (1969).

[7]V. G. Neudachin, G. A. Novoskoltseva, and Y. F. Smirnov, Zh. Eksperim. i Teor. Fiz. 55, 1039 (1968) [Sov. Phys. JETP 28, 540 (1969)].

[8]A. E. Glassgold and G. Ialongo, Phys. Rev. 175, 151 (1968).

[9]L. Vriens, Physica 47, 267 (1970).

[10]L. Vriens, Physica 49, 602 (1970).

[11]The derivation given by Eqs. (13)–(20) of Ref. 1 is, apart from minor details, equivalent to the derivation given by Eqs. (6-5-31)–(6-5-42) of Ref. 5. The sum rules (24)–(26) of Ref. 1 are equivalent to the sum rules (6-5-26)–(6-5-30) of Ref. 5, and Eqs. (6) and (10) of Ref. 1 apply also for electron scattering [see Eqs. (6-5-1)–(6-5-3) of Ref. 5].

[12]P. M. Platzman and N. Tzoar, Phys. Rev. B 2, 3556 (1970).

[13]Y. K. Kim and M. Inokuti, Phys. Rev. 165, 39 (1968). The correlation terms in Eq. (2) are negligible for large K.

[14]M. Inokuti, Rev. Mod. Phys. (to be published). See Eqs. (4.81)–(4.83).

[15]N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon, Oxford, England, 1965), p. 301.

[16]H. Ehrhardt, M. Schulz, T. Tekaat, and K. Willman, Phys. Rev. Letters 22, 89 (1969).

[17]J. Kessler, Rev. Mod. Phys. 41, 3 (1969).

[18]C. E. Kuyatt and J. A. Simpson, Rev. Sci. Instr. 38, 103 (1967), and references therein.

[19]A. M. Bincer, Phys. Rev. 107, 1434 (1957).

[20]A. T. Stewart and L. O. Roellig, *Positron Annihilation* (Academic, New York, 1967), and references therein.

[21]P. E. Mijnarends, Phys. Rev. 160, 512 (1967).

[22]L. A. Vredevoe and R. E. de Wames, Phys. Rev. 176, 684 (1968).

[23]H. Raether, *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1965), Vol. 38, p. 84.

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Ultrasonic Attenuation near the Spin-Flip Temperature in Chromium

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Measurements of the attenuation coefficient of longitudinal ultrasonic waves have been made on a single crystal of chromium near the spin-flip order-order transition. Measurements were made for sound propagation along a ⟨100⟩ axis with the crystal in the normal state and after magnetic field cooling through the Néel point in two different geometries. Considerable differences in behavior were found between the different magnetic geometries and also upon applying a large magnetic field near the spin-flip temperature. A mechanism involving switching of the polarization of the spin density waves at domain boundaries is proposed to account for the results. The relaxation time associated with this switching is estimated to be 10^{-10} sec.

INTRODUCTION

It has been recognized in the last few years that ultrasonic attenuation studies near phase transitions can give valuable information about phase-change mechanisms as well as enable the determination of the critical indices which characterize the transitions.[1–5] Of particular interest in this connection is the study of chromium, since it is now well established that below 312 °K chromium is an antiferromagnet of the spin-density-wave type. Between about 122 and 312 °K the spin density waves are transversely polarized. As chromium is cooled through the spin-flip temperature T_F at 122 °K, it undergoes a first-order phase transition, and the spin polarization changes from transverse to longitudinal. The variation of the sound attenuation near the Néel temperature T_N was previously studied by O'Brien and Franklin[6] and by Luthi, Moran, and Pollina,[7] who observed a large peak in the absorption at T_N. We have performed similar measurements near the order-order transition at T_F and report the results in this paper.

EXPERIMENTAL PROCEDURE

The measurements were made on two polycrystalline samples of lengths 0.941 and 1.010 cm, and one single crystal of chromium of length 0.815 cm. The polycrystalline samples were spark cut from a bar supplied by MRC and annealed in a vacuum of better than 10^{-6} mm Hg for about 10 h at 1250 °C. A grain structure etch showed that the samples consisted of several grains from 1 to 5 mm in diameter. The resistance ratios of the samples after annealing were both 130. The single-crystal sample, prepared from a bar supplied by Aremco, was spark